Indian Statistical Institute, Bangalore
B. Math (Hons.) Third Year
First Semester - Combinatorics and graph theory
Back paper Exam   Date: January 04, 2019
Maximum marks: 100   Duration: 3 hours

1. Let $d \geq 3$ be an integer. Then show that there exists a Hadamard matrix of order $d+1$ iff in R^d a regular simplex S can be inscribed in a hypercube C in such a way that every vertex of S is a vertex of C.   [20]

2. Let D be a $2-(v,k,\lambda)$ design with parameters $\vartheta = \binom{k+1}{2}, \lambda = 2$. Then show that any two distinct blocks of D have either 1 or 2 common points. [15]

3. Let $q \equiv 7 \pmod{16}$ be a prime power, and E be the Paley-Hadamard 2-design on q points. Then show that all the odd weights occuring in the binary code of E are $\geq \sqrt{q}$.   [20]

4. Let G be a finite bipartite graph with at least two vertices in each part. Suppose there is a constant $c \geq 2$ such that any two vertices $x \neq y$ of G at even distance have exactly c common neighbours. Let d denote the graphical distance on G.

   (a) Count in two ways the shortest paths joining x to y to show that $d(x,y) = 3 \Rightarrow \deg(x) = \deg(y)$.

   (b) Count in two ways the 4-cycles in G to show that both parts of G have the same size, say w.

   (c) Count in two ways the paths of length 3 from x to y to show that $d(x,y) = 1 \Rightarrow \deg(x) = \deg(y)$.

   (d) Hence show that G is regular, say of degree r.

   (e) Count in two ways the paths of length 2 in G to show that $w = 1 + \frac{r(r-1)}{c}$.

   (f) If w is even then show that $r - c$ must be a perfect square.   [30]

5. Let H be the subgroup of order 9 in the multiplication group of the field of order 73. Show that the additive translates of H are the lines of a projective plane of order 8.   [15]